PHYSICS - I
(PHYS 1001)

Time Allotted : 3 hrs **Full Marks : 70**

Figures out of the right margin indicate full marks.

Candidates are required to answer Group A and any 5 (five) from Group B to E, taking at least one from each group.

Candidates are required to give answer in their own words as far as practicable.

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternative for the following: **10 × 1 = 10**

(i) If for any vector field $\vec{F}$, $\vec{\nabla} \times \vec{F} = \alpha\vec{F}$ the vector field is
(a) source (b) sink
(c) irroational (d) solenoidal.

(ii) A necessary condition for a vector field $\vec{v} = f(x,y)\hat{\imath} + g(x,y)\hat{\jmath}$ to be conservative is
(a) $\frac{\partial f}{\partial x} = \frac{\partial g}{\partial y}$ (b) $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$ (c) $\frac{\partial f}{\partial x} = -\frac{\partial g}{\partial y}$ (d) $\frac{\partial f}{\partial y} = -\frac{\partial g}{\partial x}$.

(iii) Which of the following quantities is independent of time for a particle moving under the influence of a central force?
(a) Linear velocity (b) Radial velocity
(c) Areal velocity (d) Linear acceleration.

(iv) The potential energy of a particle executing SHM of amplitude a is equal to its kinetic energy when displacement of the particle is
(a) $\pm a$ (b) $\pm\frac{a}{\sqrt{2}}$ (c) $\pm\frac{a}{2}$ (d) $\pm\frac{a}{4}$.

(v) The curve representing the relation between relaxation time and damping constant for a weakly damped oscillator is
(a) parabola (b) hyperbola
(c) straight line (d) ellipse.

(vi) A polarized light given by the light vector $\vec{E}(z,t) = 0.2\,\{\cos(\omega t - kz)\,\hat{\imath} + \sin(\omega t - kz)\,\hat{\jmath}\}$ is passed through a half wave plate with optic axis parallel to x-axis. The light will be
(a) circularly polarized (b) linearly polarized
(c) elliptically polarized (d) plane polarized.

(vii) A moving charge produces
(a) $\vec{E}$ field only (b) $\vec{B}$ field only
(c) both $\vec{E}$ and $\vec{B}$ field (d) none of these.

(viii) A time varying magnetic field $\vec{B} = B_0\cos(2z - \omega t)\,\hat{\imath}$ is producing an electric field $\vec{E} = E_0\cos(2z - \omega t)\hat{\jmath}$. Then the magnitude of ω is
(a) $2\frac{E_0}{B_o}$ (b) $2\frac{B_0}{E_0}$ (c) $\frac{B_0}{E_o}$ (d) $\frac{E_0}{B_o}$.

(ix) The differential form of Faraday's law of electro-magnetic induction is
(a) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ (b) $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$ (c) $\vec{\nabla}\times\vec{B} = -\frac{\partial\vec{E}}{\partial t}$ (d) $\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$

(x) The intrinsic magnetic moment of the atoms of a material is not zero. The material
(a) must be paramagnetic
(b) must be diamagnetic
(c) must be ferromagnetic
(d) may be paramagnetic or ferromagnetic

Group – B

2. (a) Show that $\vec{\nabla}.(\vec{\nabla}\times\vec{A})$ =0 always, where $\vec{A}$ is a vector field.

(b) (i) Prove that $\vec{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x,y,z) = c$, where c is a constant.

(ii) If $\vec{V} = (x+2y+az)\hat{i} + (bx-3y-z)\hat{j} + (4x+cy+2z)\hat{k}$ Find the constants a, b, c so that $\vec{\nabla}\times\vec{V} = 0$.

(c) (i) Show that $\vec{\nabla}.(\frac{\vec{r}}{r^3}) = 0$ where $\vec{r}$ is the position vector (excluding the origin).

(ii) If a point is represented by the following parametric equations $x = 2t$ and $y = 19 - 2t^2$, at what time velocity is perpendicular to its acceleration?

2 + (2 + 3) + (3 + 2) = 12

3. (a) A particle rotating with constant angular velocity $\vec{\omega}$ has the linear velocity $\vec{v}$. Show that $2\vec{\omega}$=$\vec{\nabla}\times\vec{v}$.

(b) For a particle moving under the influence of a central force field, show that, (i) the total energy remain conserved (ii) upon the withdrawal of the force, it starts to follow a rectilinear path.

(c) Equation of an orbit of a particle under the action of central force is $r = ae^{b\theta}$, Find the corresponding force.

(d) What is the effect of Coriolis force on a particle falling freely under the action of gravity. Comment on the equation $\vec{g}_{eff} = \vec{g} - \vec{\omega}\times(\vec{\omega}\times\vec{r})$.

2 + (3 + 2) + 2 + (2 + 1) = 12

Group – C

4. (a) A pendulum is performing damped oscillation follows the equation $\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + 0.09x = 0$.
   (i) Find out the condition of weakly damped oscillation.
   (ii) If at t = 0 the pendulum is kicked at the origin with a velocity v_0 find out its displacement at any time t for $\beta = 0.5$.
   (iii) Calculate the quality factor of the system for $\beta = 0.2$.

   (b) Show that the function $f(x,t) = (kx + \omega t - 1)e^{(kx+\omega t)}$ represents a classical wave. Find the wave velocity.

(2 + 3 + 2) + (4 + 1) = 12

5. (a) The equation of a forced oscillation is given by
$$\frac{d^2x}{dt^2} + 0.6\frac{dx}{dt} + 0.09x = 3\sin\omega t.$$
   (i) Find the amplitude of steady-state oscillation if $\omega = 0.2$.
   (ii) Find out the frequency corresponding to amplitude resonance.
   (iii) Calculate the average power absorbed due to damping in one complete cycle of steady state in case of resonance.

   (b) (i) Explain briefly the phenomenon of population inversion.
   (ii) An ordinary ray passes through a half wave plate made of positive crystal of refractive index 1.9. If the minimum thickness of the plate is twice as much the wave length of light used find the refractive index of extraordinary ray.

(3 + 2 + 3) + (2 + 2) = 12

Group – D

6. (a) (i) Two point charges are located at the points of intersection of the straight line $2x + 3y = 6$ and coordinate axes so that the potential at the origin is zero. Find out the ratio of their charges magnitude. If the charge on the x-axis is given by $+q$ find the electric field at the origin.
   (ii) Show that work done by an electrostatic field over a closed loop is zero.

   (b) Consider a cylindrical charge distribution having a circular cross section of radius a and charge density $\rho(r) = \rho_0(1 + \frac{r^2}{a^2})$ Find the electric field at any internal point.

(3 + 3 + 2) + 4 = 12

7. (a) Two concentric spheres of radii $2\ cm.$ and $5\ cm.$ are kept at potential V and $2V$.
   (i) Write down the relevant Laplace equation for the potential in a region $2\ cm < r < 5\ cm$.
   (ii) Find out the potential in between the spherical shells as function of radius.
   (iii) Find out the magnitude and direction of the electric field at $r = 3\ cm$.

   (b) Find the electric field $\vec{E}$ at any point (r,θ), where potential is $\phi(r,\theta) = r^2\cos\theta$.

   (c) Show that potential due to a dipole placed at the origin at a large distance is given by, $\phi(r,\theta) = \frac{\vec{p}.\vec{r}}{4\pi\varepsilon_0 r^3}$, where $\vec{p}$ is the dipole moment and (r,θ) is the coordinate of the point where the potential has to be found out.

(1 + 4 + 2) + 2 + 3 = 12

Group – E

8. (a) Using Biot-Savart law obtain the magnetostatic field produced at the centre of a circular coil of radius a due to a current i in it.

   (b) (i) Show that a classical magnetic field always admits a vector potential.
   (ii) A current i is flowing through a wire of length $2L$ along the positive direction of Y-axis with X-axis being the perpendicular bisector. Calculate the vector potential at any point on the X-axis. Calculate the magnetic field at $(L, 0)$.

4 + (2 + 3 + 3) = 12

9. (a) Show that the magnetic field $\vec{B}$ and its corresponding vector potential $\vec{A}$ are related by the equation $\oint \vec{A}.d\vec{l} = \int \vec{B}.d\vec{s}$.

   (b) Obtain the differential form of Faraday's law using Stoke's theorem.

   (c) A time varying magnetic field $\vec{B} = B_0\sin\omega t(\hat{j} + \hat{k})$ is allowed to pass through a square loop of area $a^2(\hat{\imath}+\hat{k})$. Find the maximum value of induced e. m. f. in the loop.

   (d) Differentiate among dia, para and ferromagnetic materials in terms of their behaviour in the magnetic field.

3 + 3 + 3 + 3 = 12